9/17/19

MLS3: Life Tables & Common Mortality Models (Continued)

I) DML (ω)

Note: $_tP_x = \dfrac{\omega - x - t}{\omega - x}$

II) Generalized DeMoivre's Law (Beta Distribution)

GDML (ω, α)

$$_tP_x = \left(\frac{\omega - x - t}{\omega - x}\right)^{\alpha}$$

$$\mu_{x+t} = \frac{\alpha}{\omega - x - t}$$

$$\mathring{e}_x = \frac{\omega - x}{\alpha + 1}$$

Remark: GDML$(\omega, \alpha = 1)$ = DML(ω)

III) Constant Force (of Mortality)

CF(μ) : $X \sim \text{Exp}\left(\text{mean} = \frac{1}{\mu}\right)$

Fact: $\left(T_x = X - x \mid X > x\right) \sim \text{Exp}\left(\text{mean} = \frac{1}{\mu}\right)$

$\therefore \quad {}_tP_x = e^{-\mu \cdot t} \rightsquigarrow P_x = e^{-\mu} = p$

$\therefore \quad {}_tP_x = p^t$

$$_tq_x = 1 - e^{-\mu \cdot t} = 1 - p^t$$

III) (Continued) $CF(\mu)$

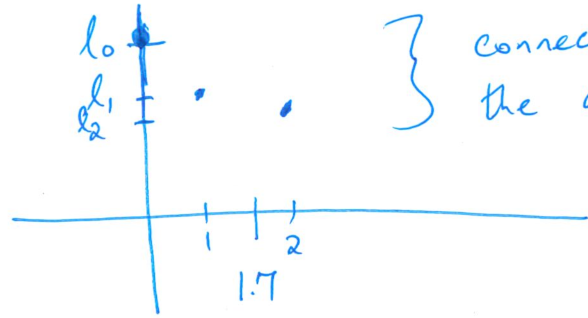
$$f_x(t) = \mu \cdot e^{-\mu \cdot t}$$

$$\left.\begin{aligned} \mathring{e}_x &= \frac{1}{\mu} \\ Var(T_x) &= \frac{1}{\mu^2} \end{aligned}\right\} E[T_x^2] = \frac{2}{\mu^2}$$

<u>Remark</u>: None of these functions depend on x.

MIS4: Extending the Discrete Framework to a Continuous Framework

IDEA: We have l_x values for integer ages from the SULT. We want to extend to l_x values for all x.

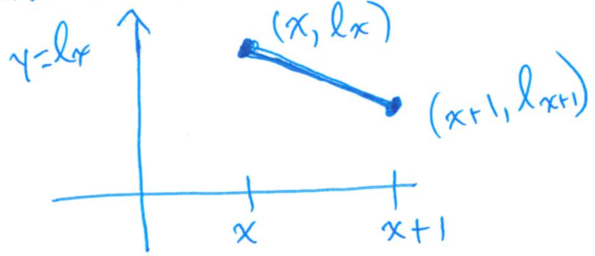


} connect the dots

$l_{1.7}$

<u>2 Cases</u>:

Case 1: Uniform Distribution of Deaths (UDD)



use straight line

Q: ? l_{x+t} when $0 \le t \le 1$

slope $m = \dfrac{l_{x+1} - l_x}{1}$ & use point $(x, \overset{y_1}{l_x})$

and $(x+t, \overset{y}{l_{x+t}})$ $\qquad y - y_1 = m(x - x_1)$

$$l_{x+t} - l_x = (l_{x+1} - l_x)\cdot t$$

$$\therefore l_{x+t} = (1-t)\cdot l_x + t\cdot l_{x+1}$$

Fact: $l_{x+t} \overset{UDD}{=}$ weighted (arithmetic) average of l_x & l_{x+1}

E.g. $l_{47.2} = .8\cdot l_{47} + .2\, l_{48}$

$$\therefore {}_{17.2}q_{30} = \frac{l_{30} - l_{47.2}}{l_{30}} \overset{UDD}{=} \frac{l_{30} - (.8 l_{47} + .2 l_{48})}{l_{30}}$$

Test 3 Review!

1) Life Table Notation $\qquad$ 1(b) & 1(d) MIS3

Recall $\quad {}_{k|n}q_x = \dfrac{l_{x+k} - l_{x+k+n}}{l_x}$

2) MIS2: 19–25